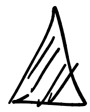
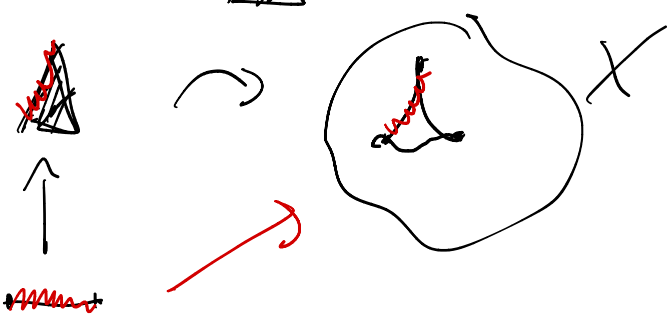
n-simplessi standard

I intervento: $\mathrm{Sing}(X)$ (X spazio topologico)

$$\mathrm{Sing}_n(X) = \{\text{n-simplessi di } \mathrm{Sing}(X)\}$$
$$= \{\text{funzioni continue } |\Delta^n| \to X\}$$

"n-simplesso (geometrico)"

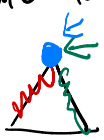
$|\Delta^2|$:  $|\Delta^1|$: $|\Delta^0|$: •



<u>Definizione</u>: un <u>insieme simpliciale</u> è una famiglia di insiemi $\{X_n : n \geq 0\}$ con funzioni

$$d^i: X_n \to X_{n-1} \quad (\text{facce})$$
$$s^i: X_n \to X_{n+1} \quad (\text{degenerazioni})$$

soggette ad opportune relazioni (IDENTITÀ SIMPLICIALI)

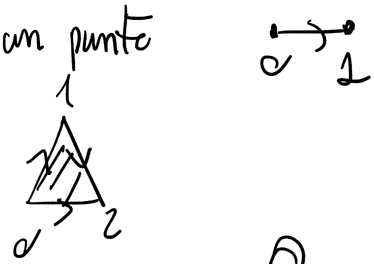


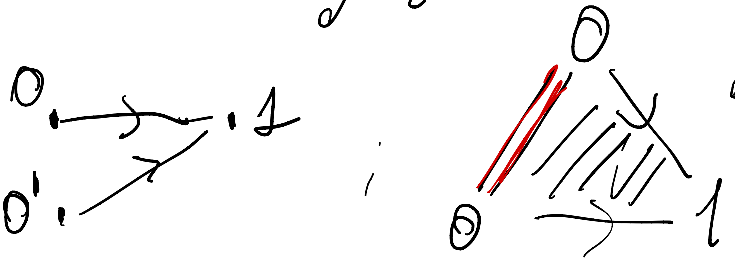
$\boxed{X_n: \text{ n-simplessi di } X}$

Un po' di euristica:

$\bullet$ = un punto

0





un "triangolo degenerato"

Simplessi standard!

X

X_n: n-simplessi

X_0: punti

X_1: segmenti orientati

$\vdots$

Δ^0: $\bullet$ 0

Δ^1: $0 \to 1$

Δ^2:

Δ^3: $\ldots$ Δ^n:

$(\Delta^2)_2$? $(\Delta^4)_2$! $(\Delta^3)_0$?

$\underline{n=0}$. Punto: $\boxed{0}$

$\underline{n=1}$: $\boxed{0 < 1}$

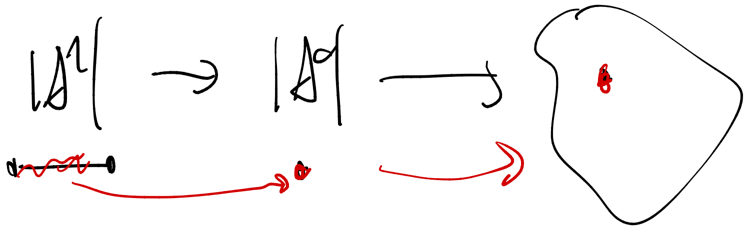
$\underline{n=2}$: $\boxed{0 < 1 < 2}$

L'intuizione: $0 \to 1$

$$\delta^1 : \{0\} \longrightarrow \{0 < 1\}$$
$$0 \longmapsto 0$$

"seleziona il punto 0 in $0 \to 1$".

$$\left(|\Delta^1| \to |\Delta^0| \longrightarrow X \right)$$



$$\delta^0 : \{0\} \longrightarrow \{0<1\}$$
$$0 \longmapsto 1$$

$\delta^0, \delta^1, \sigma^0$ sono NONDECRESCENTI

$$\sigma^0 : \{0<1\} \longrightarrow \{0\}$$
$$0 \longmapsto 0$$
$$1 \longmapsto 0$$

Cosa succede con $n=2$: $\{0<1<2\}$

$$\delta^0 : \{0<1\} \longrightarrow \{0<1<2\}$$

$$0 \longmapsto 1$$
$$1 \longmapsto 2.$$

$\cancel{0 \mapsto 2}$
$\cancel{1 \mapsto 1}$

$$\delta^1 : \{0<1\} \longrightarrow \{0<1<2\}$$

$$0 \longmapsto 0$$
$$1 \longmapsto 2$$

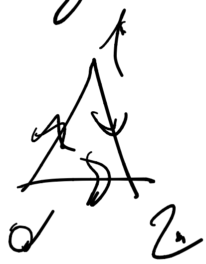
$$\delta^2 : \{0<1\} \longrightarrow \{0<1<2\}$$

$$0 \longmapsto 0$$
$$1 \longmapsto 1$$

$$\sigma^0 : \{0<1<2\} \longrightarrow \{0<1\}$$

"Collasso 0 e 1":

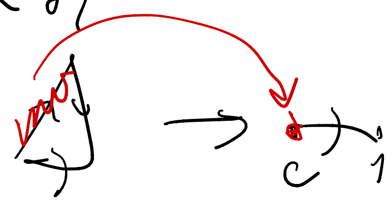
$$0 \longmapsto 0$$
$$1 \longmapsto 0$$
$$2 \longmapsto 1$$



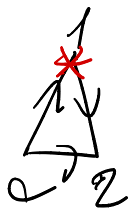
$$\sigma^1 : \{0<1<2\} \longrightarrow \{0<1\}$$

"collasso 1 e 2":

$$0 \longmapsto 0$$
$$1 \longmapsto 1$$
$$2 \longmapsto 1$$



$$\{0\} \longrightarrow \{0<1<2\}$$



$$0 \longmapsto 1$$

$$[n] = \{0 < \dots < n\} \sim \Delta^n$$

$\left(\mathrm{Sing}_k(X) = \{\text{funzioni continue } |\Delta^k| \to X\} \right)$

$\uparrow$ k-simplesso geometrico $\quad \left(\nwarrow |\Delta^{k-1}| \right)$

$\boxed{\begin{aligned} \Delta^n_k &= \text{k-simplessi di } \Delta^n \\ &= \{\text{funzioni nondecrescenti } [k] \to [n]\} \\ &\qquad \{0<\dots<k\} \to \{0<\dots<n\} \end{aligned}}$

Chi sono $d_i : \Delta^n_k \to \Delta^n_{k-1}$?

$$\left(\begin{array}{ccc} |\Delta^k| & \longrightarrow & X \\ \uparrow \delta^i & & \\ |\Delta^{k-1}| & & \end{array} \right)$$

$\sigma \in (\Delta^n)_k$

$$\begin{tikzcd} {[k]} \arrow[r, "\sigma"] & {[n]} \\ {[k-1]} \arrow[u, "\delta^i"] \arrow[ur] & \end{tikzcd}$$

$\sigma \circ \delta^i = d_i(\sigma)$

$\delta^i : [k-1] \longrightarrow [k]$

$$\boxed{\begin{array}{ccc} 0 & \longmapsto & 0 \\ \vdots & & \\ i-1 & \longmapsto & i-1 \\ i & \longmapsto & i+1 \\ \vdots & & \\ k-1 & \longmapsto & k \end{array}}$$

$\sigma^i : [k+1] \to [n]$

$\{0<\dots<k+1\} \to \{0<\dots<k\}$

$$\begin{tikzcd} {[k]} \arrow[r, "\tau"] & {[n]} \\ {[k+1]} \arrow[u, "\sigma^i"] \arrow[ur, "\tau\circ\sigma^i"'] & \end{tikzcd}$$

$= s_i(\tau)$

$\{\Delta^n_k : k \geq 0\}$ con d_i, s_i, è un <u>insieme simpliciale</u>, denotato con Δ^n, detto "n-simplesso standard",

Δ^0: Δ^0_0: funzioni nondecr. $[0] \to [0]$

$\{0\} \to \{0\}$

(ne ho una sola!)

$\bullet$

0

Δ^0_1: funzioni nondecrescenti $[1] \to [0]$

$\{0<1\} \to \{0\}$

$0 \mapsto 0$

$1 \mapsto 0$

Δ^0_k: funzioni nondecr. $[k] \to [0]$

$0 \mapsto 0$

$\vdots$

$k \mapsto 0$